Homework #6 (100 points) - Show all work on the following problems:

(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (30 points): This problem looks at the properties of EM waves in conductors.

1a (10 points): Show that the skin depth $1/k_i$ in a poor conductor ($\sigma \ll \omega\varepsilon$) is $\frac{2}{\sigma}\sqrt{\varepsilon/\mu}$ regardless of frequency.

1b (10 points): Show that the skin depth $1/k_i$ in a good conductor ($\sigma \gg \omega\varepsilon$) is $\lambda/(2\pi)$.

1c (10 points): Show that in a good conductor the magnetic field lags the electric field by 45°.

Problem 2 (30 points): Start with the real form for EM plane waves in a conductor:

$$\vec{E}(z,t) = E_0 e^{-k_i z}\cos(k_r z - \omega t + \delta)\,\hat{x}$$
$$\vec{B}(z,t) = B_0 e^{-k_i z}\cos(k_r z - \omega t + \delta + \phi)\,\hat{y}$$

2a (20 points): Show that the time-averaged energy density is $\langle u\rangle = \frac{{k_r}^2}{2\mu\omega^2}{E_0}^2 e^{-2k_i z}$, and show that the magnetic portion of the energy always dominates (unlike in vacuum or in a dielectric, where they are equal).

2b (10 points): Show that the intensity (time-averaged energy flux) of the light in the conductor is $I = \frac{k_r}{2\mu\omega}{E_0}^2 e^{-2k_i z}$.

Problem 3 (40 points):

3a (20 points): Show directly that the solutions for a guided wave in a coaxial cable satisfy all four Maxwell's equations and the relevant boundary conditions at the inner and outer radii.

$$\vec{E}(s,\phi,z) = \frac{E_0}{s}\cos(kz - \omega t)\,\hat{s} \qquad \vec{B}(s,\phi,z) = \frac{E_0}{cs}\cos(kz - \omega t)\,\hat{\phi}$$

3b (20 points): Find the total linear charge density $\lambda(z,t)$ and the total current $I(z,t)$ on the surface of the inner conductor.